From Brownian Motor to Brownian Refrigerator

- Can thermal fluctuations reduce themselves? -

Ryoichi Kawai

Department of Physics
University of Alabama at Birmingham

Solvay Institute, March 22, 2006

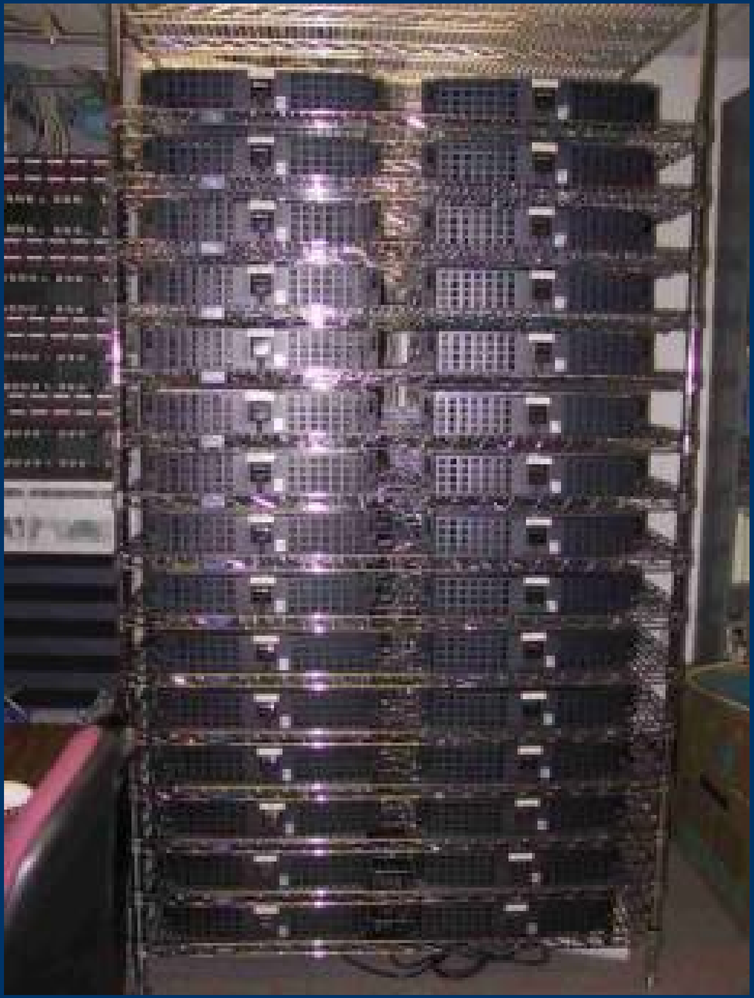
Collaboration

Chris Van den Broeck
(University of Hasselt)

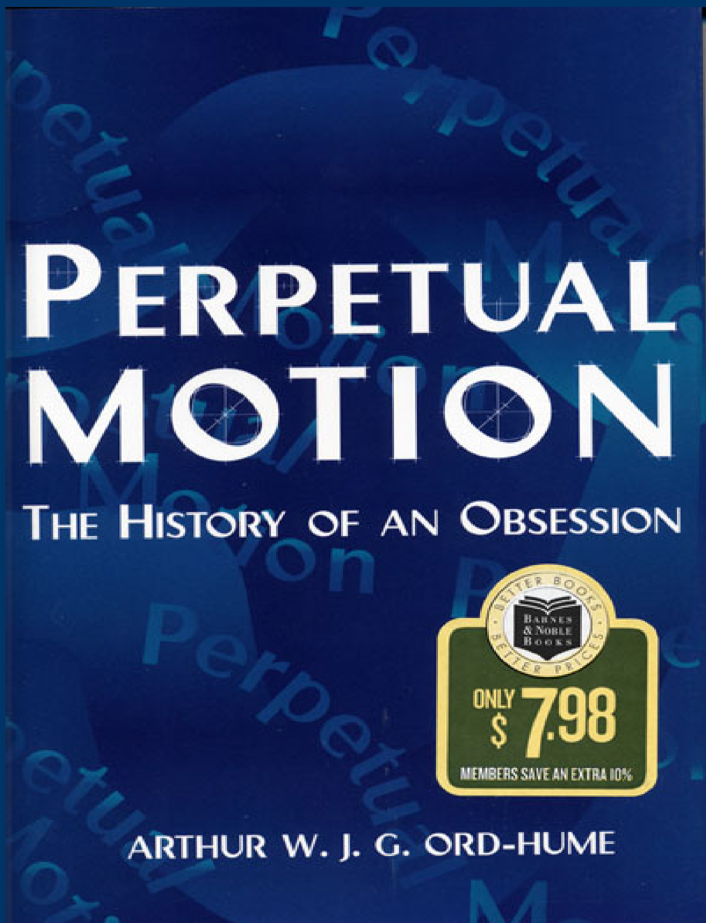
Phys. Rev. Lett. 93 (2004), 090601

New J. Phys. 7 (2005), 10

Phys. Rev. Lett. 96 (2006), 210601



Beowulf cluster



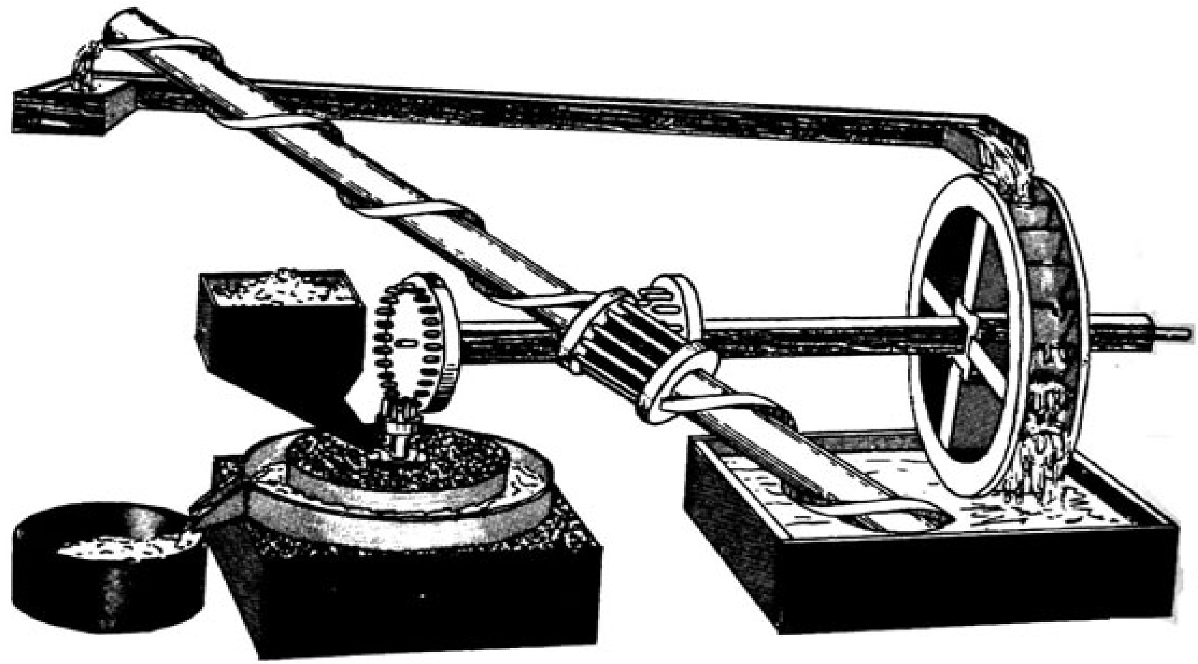


Fig. 7. Closed-cycle mill proposed by Robert Fludd in 1618. Perpetual power was offered by this means for areas where running streams did not exist. Not until two centuries after Fludd's death was it understood that energy conservation made this sort of thing impossible.

Smoluchowski-Feynman Ratchet

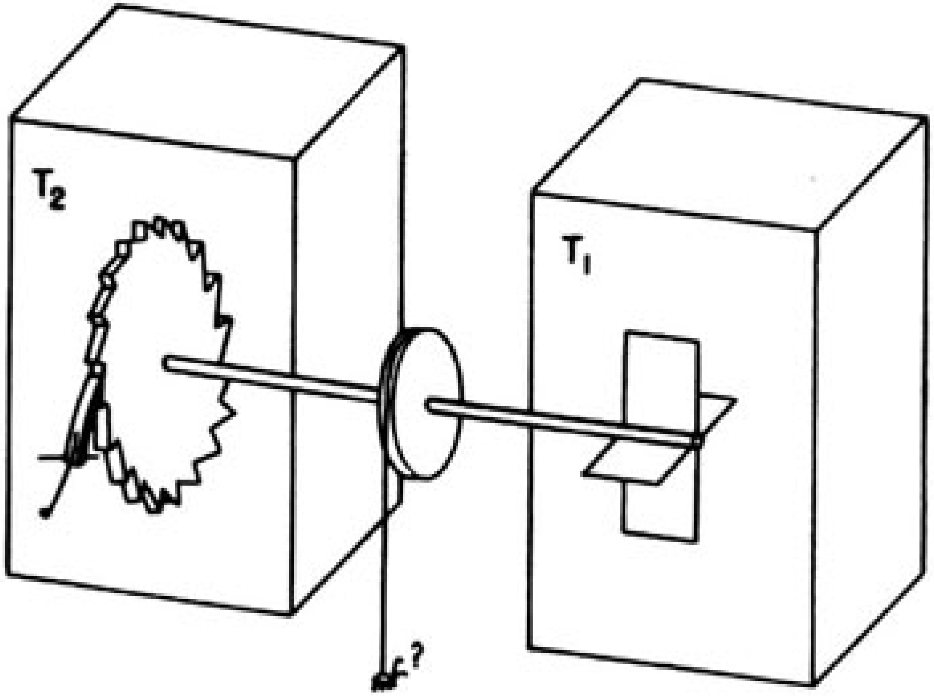
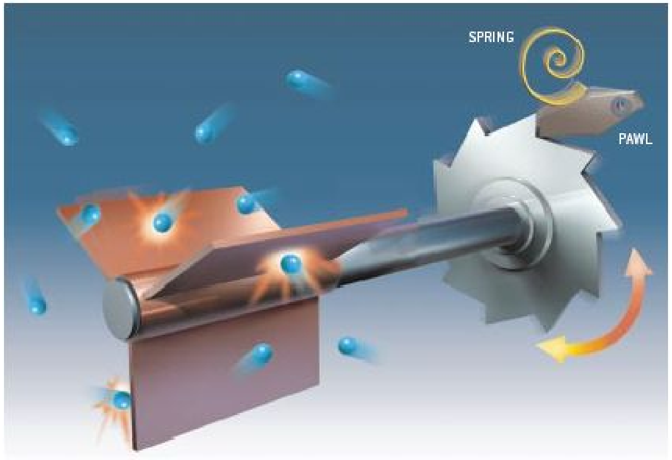


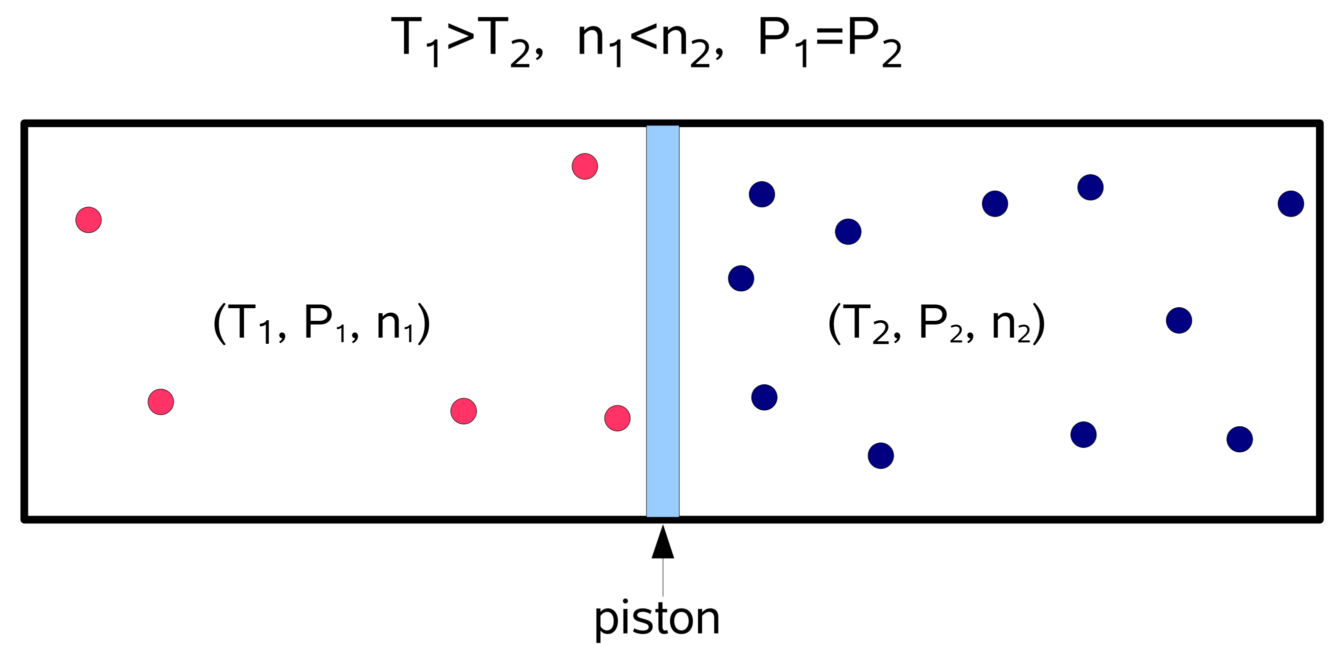
Fig. 46–1. The ratchet and pawl machine.



The Feynman Lecture on Physics, Vol. I
(Feynman, Leighton, & Sands, 1963)

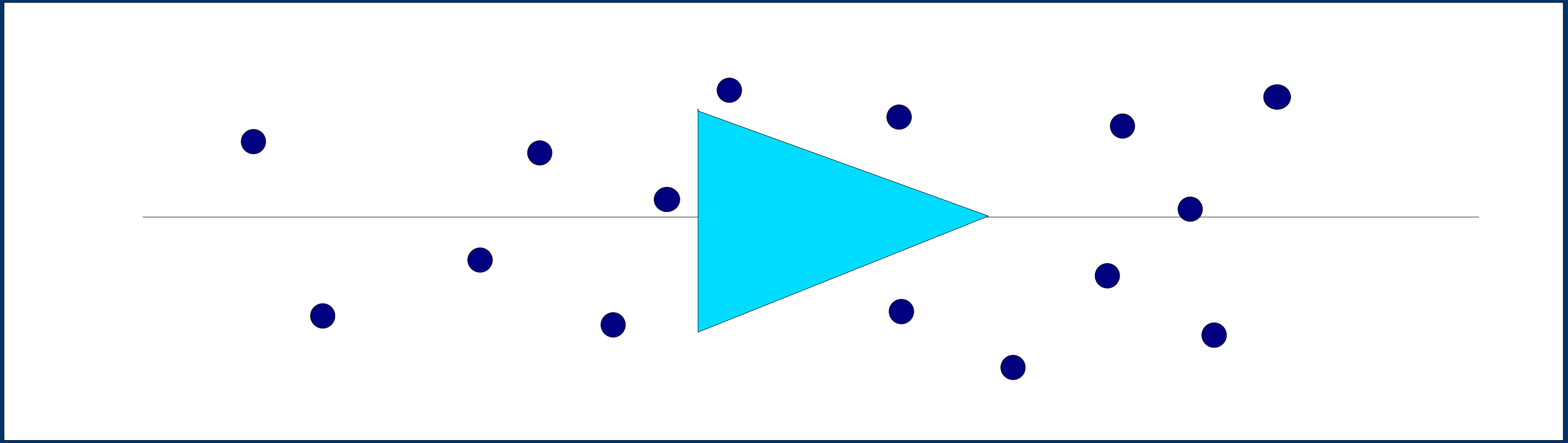
D. Astumian, Scientific American
July, 2001

Adiabatic Piston



Does the piston move?

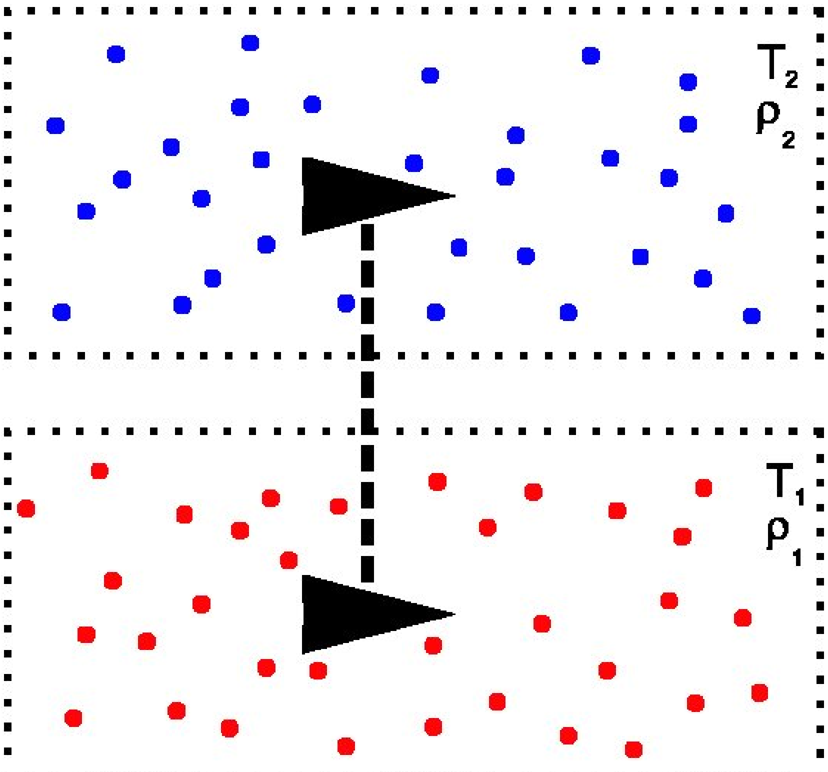
Asymmetric Object



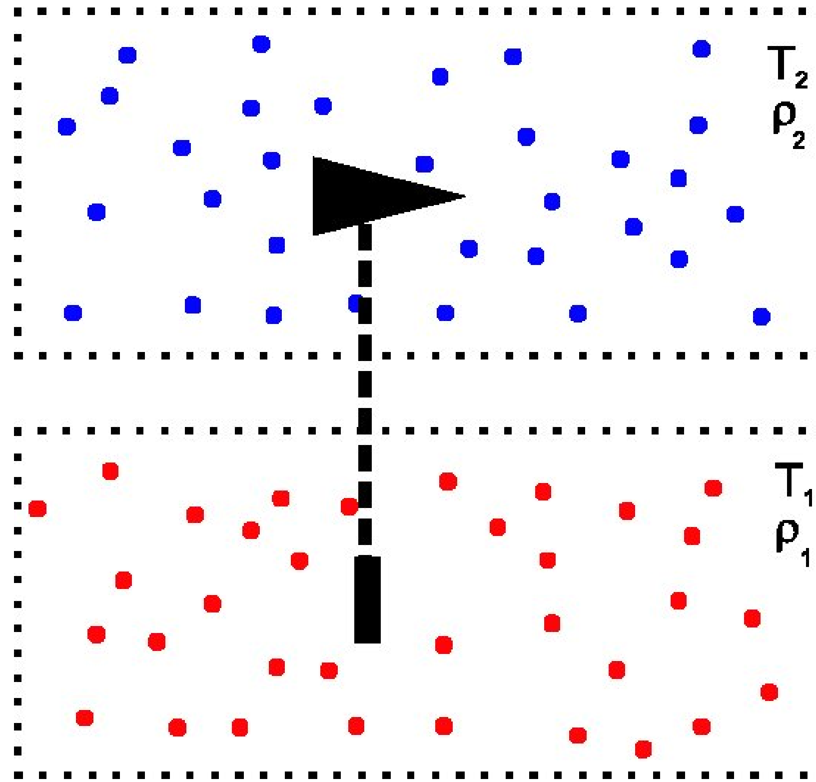
Does the triangle drift to the right?

No, it does not. (otherwise it violates 2nd Law)

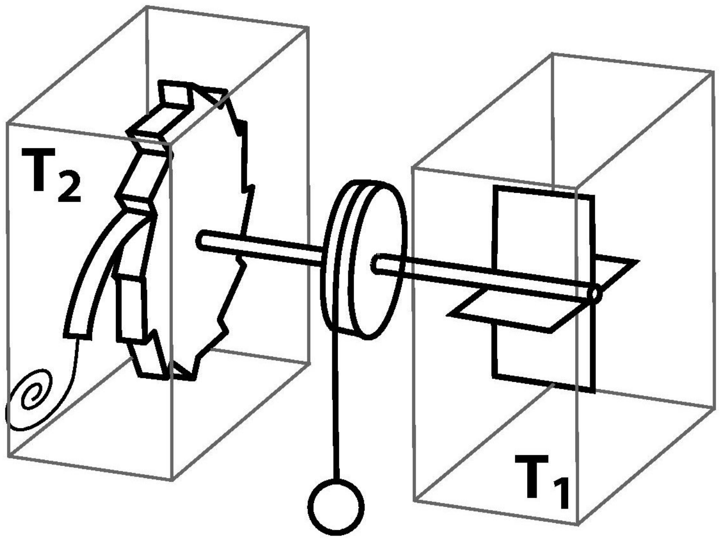
Triangula



Triangulita

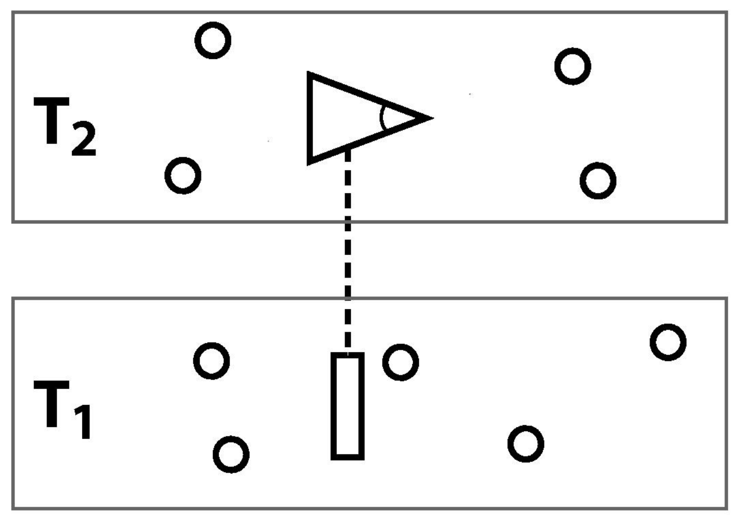


Feynman ratchet vs. Triangulita



Pawl exerts force on the system.

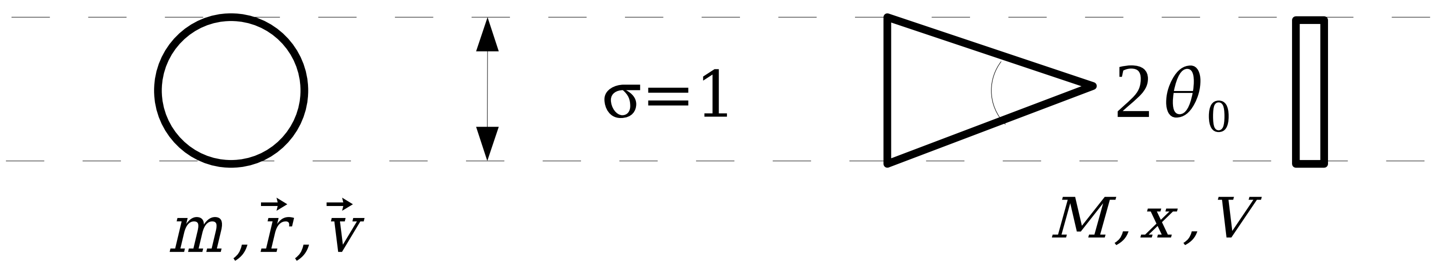
Momentum does not conserve.



Computer Simulations

Hard disk molecular dynamics simulation with advanced list management algorithms.

10,000 particles can be easily simulated on a laptop computer.

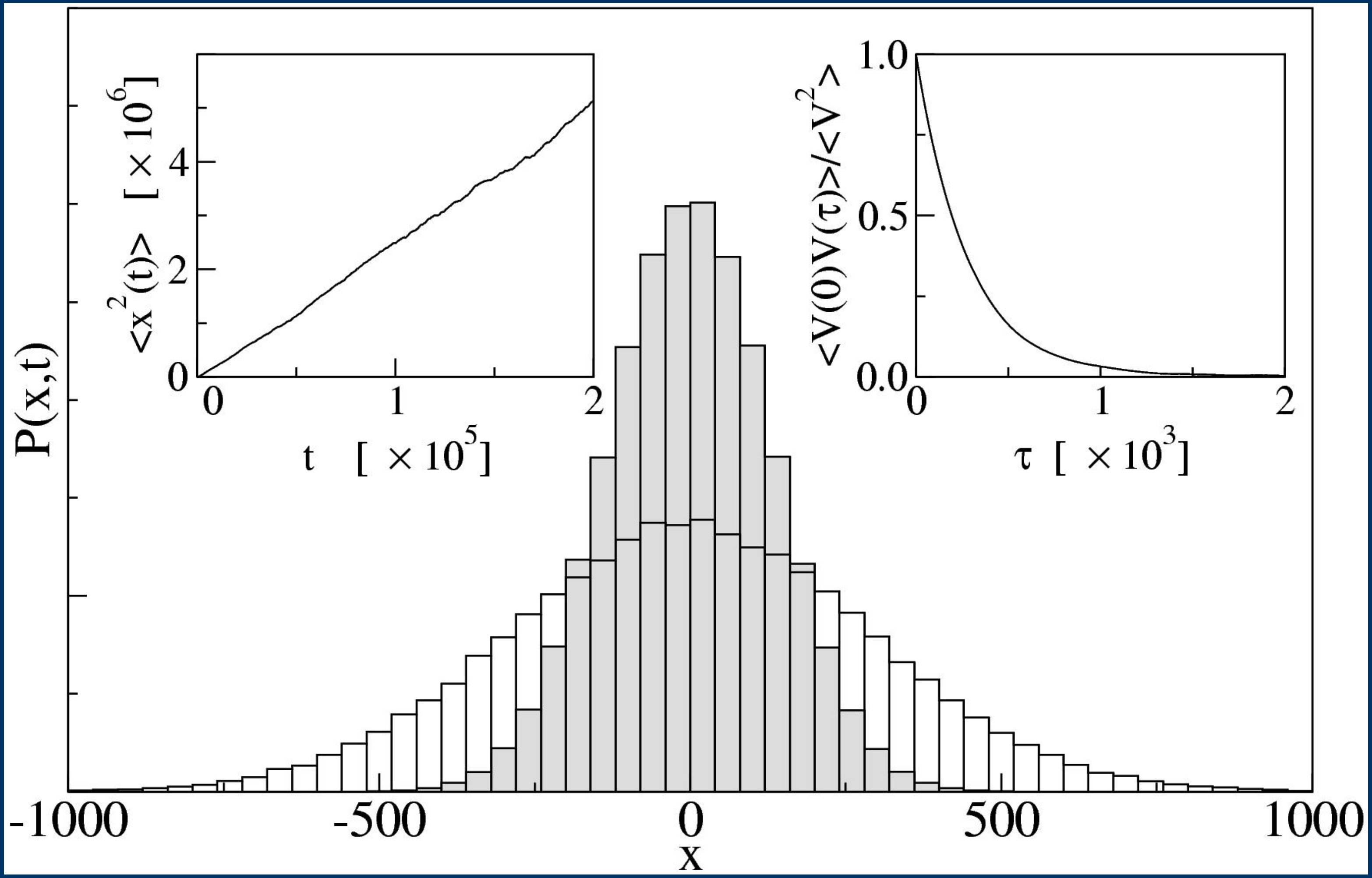


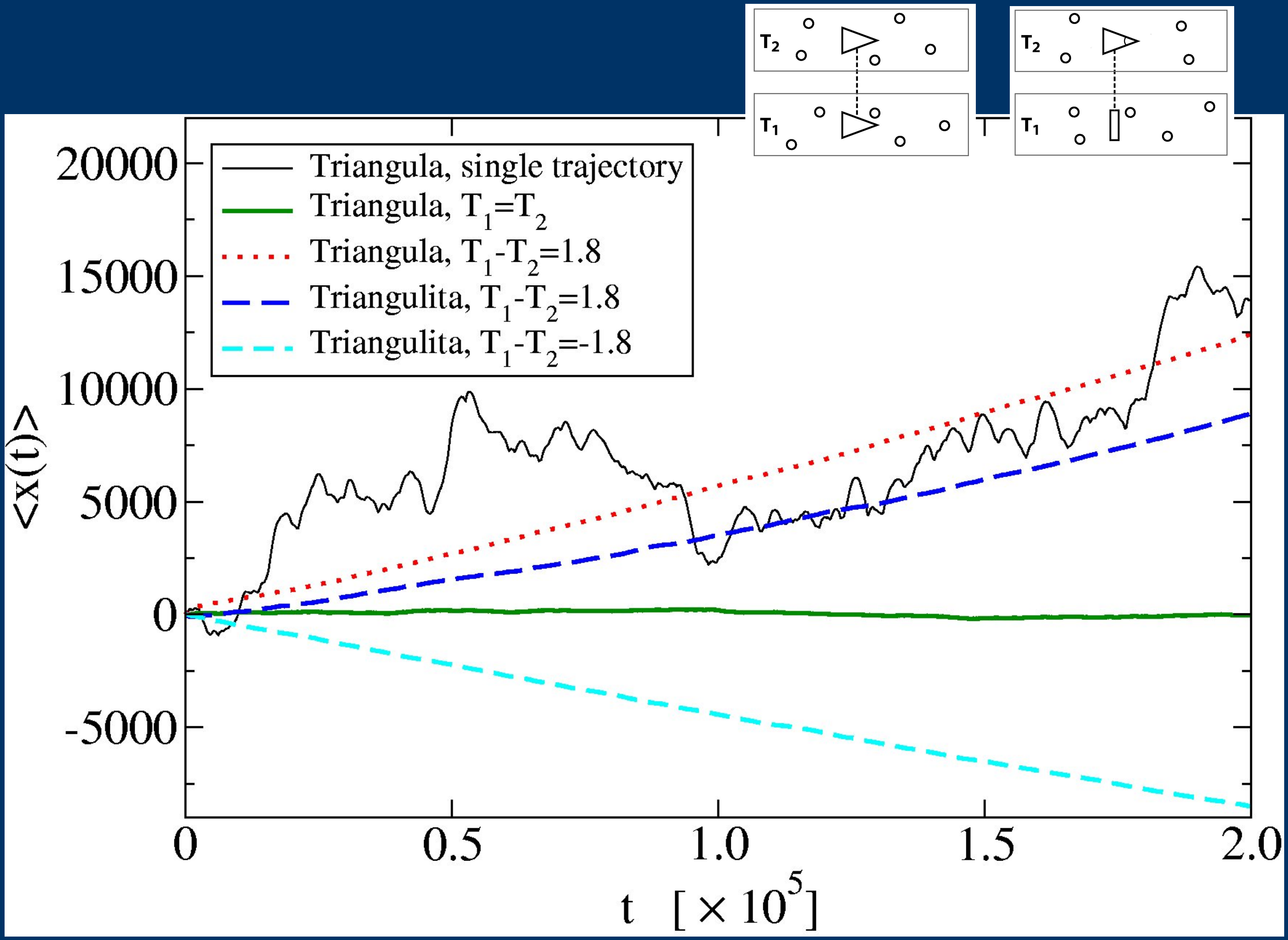
Gas particle

Brownian motor

Thermodynamic quantities: $T_i, \Omega_i, N_i, \rho_i = N_i/\Omega_i \, (i=1,2)$

$T_1=T_2$





T2
T1
T2
T1
Triangula, single trajectory
Triangula, $T_1=T_2$
Triangula, $T_1-T_2=1.8$
Triangulita, $T_1-T_2=1.8$
Triangulita, $T_1-T_2=-1.8$
20000
15000
10000
5000
0
-5000
<x(t)>
0
0.5
1.0
1.5
2.0
t [× 10^5]

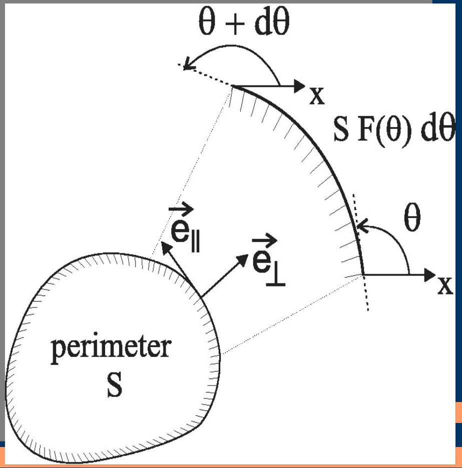
Boltzmann-Master Equation

Velocity distribution of the motor

$$\frac{\partial P(V,t)}{\partial t} = \int dV W(V|V')P(V',t) - \int dV' W(V'|V)P(V,t)$$

Transition probability from *V'* to *V*.

$$W(V,r) = \sum_i \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y$$
$$\times \rho_i \phi_i(v_x, v_y) L_i F_i(\theta)(\vec{V} - \vec{v}) \cdot \vec{e}(\theta) H[(\vec{V} - \vec{v}) \cdot \vec{e}(\theta)]$$
$$\times \delta \left[r + \frac{m}{M} B(\theta)(V - v_x + v_y \cot \theta) \right]$$



$$B(\theta) = 2M \sin^2\theta / (M + m \sin^2\theta)$$
$$\phi_i(v_x, v_y) = m \exp\left[-m(v_x^2 + v_y^2)/2k_B T_i\right]/2\pi k_B T_i$$

A perturbation expansion in $\epsilon = \sqrt{\frac{m}{M}}$

The lowest order

Liner Langevin Equation:

$$M\dot{V} = -\sum_i \gamma_i V + \sum_i \sqrt{2\gamma_i k_B T_i}\xi_i$$

Friction coefficient: $\gamma_i = 4\rho_i L_i \sqrt{\frac{m k_B T_i}{2\pi}} \int_0^{2\pi} F_i(\theta) sin^2\theta d\theta$

Gaussian white noise: $\langle \xi(t) \rangle = 0$ $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$

Solution: $\langle V \rangle = 0$

Despite of the asymmetry in the object it symmetrically diffuses without drift.

The Second Order

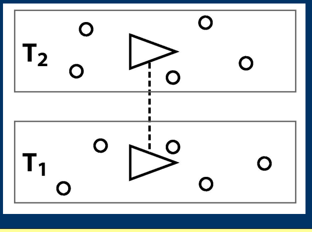
$$\langle V\rangle = \sqrt{\frac{\pi k_B T_{\text{eff}}}{8M}}\sqrt{\frac{m}{M}}\frac{\sum_i \rho_i L_i \frac{T_i - T_{\text{eff}}}{T_{\text{eff}}}\int_0^{2\pi} d\theta F_i(\theta)\sin^3\theta}{\sum_i \rho_i L_i \sqrt{\frac{T_i}{T_{\text{eff}}}}\int_0^{2\pi} d\theta F_i(\theta)\sin^2(\theta)}$$

Effective Temperature: $T_{\text{eff}} = \sum_i \gamma_i T_i \Big/ \sum_i \gamma_i$

$\langle V\rangle = 0$ if $T_1 = T_2 = T_{eff}$ or $F_i(-\theta) = F_i(\theta), \quad \forall i$

$\langle V\rangle \to 0$ when $\epsilon = \sqrt{\frac{m}{M}} \to 0$

Velocity of Triangula

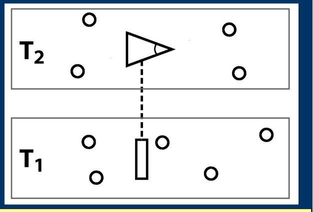


$$\langle V\rangle = \rho_1\rho_2(1-\sin\theta_0)\sqrt{\frac{m}{M}}\sqrt{\frac{\pi k_B}{8M}}\frac{(T_1-T_2)(\sqrt{T_1}-\sqrt{T_2})}{[\rho_1\sqrt{T_1}+\rho_2\sqrt{T_2}]^2}$$

$\langle V\rangle = 0$ when $T_1 = T_2$

$\langle V\rangle > 0$ when $T_1 \neq T_2$

Velocity of Triangulita

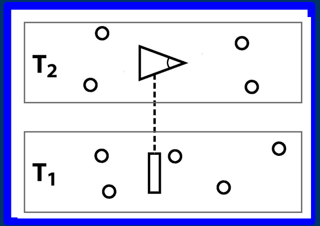


$$\langle V\rangle = \rho_1\rho_2(1-\sin^2\theta_0)\sqrt{\frac{m}{M}}\sqrt{\frac{\pi k_B}{2M}}\frac{(T_1-T_2)\sqrt{T_1}}{[2\rho_1\sqrt{T_1}+\rho_2\sqrt{T_2}(1+\sin\theta_0)]^2}$$

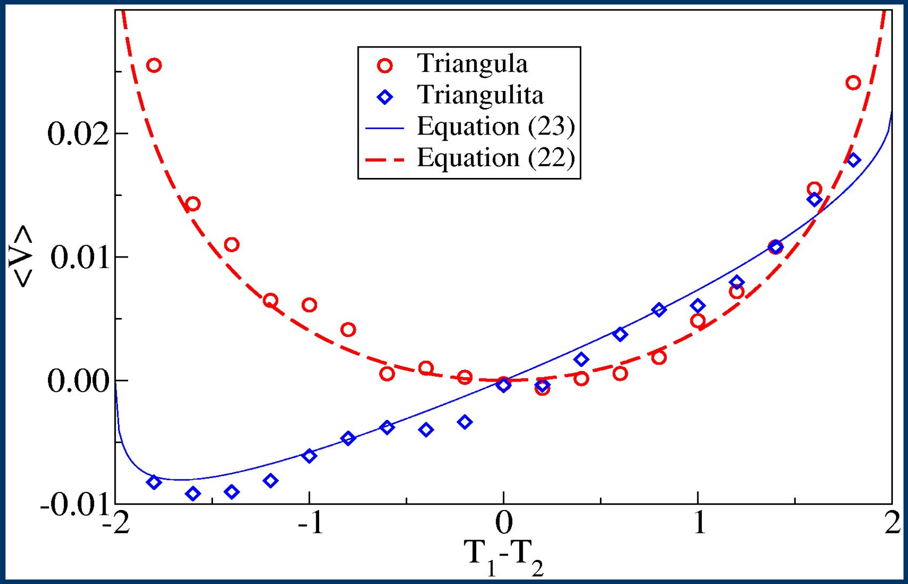
$\langle V\rangle = 0$ when $T_1 = T_2$

$\langle V\rangle > 0$ when $T_1 > T_2$

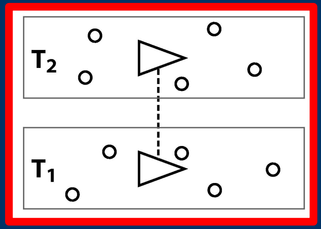
$\langle V\rangle < 0$ when $T_1 < T_2$



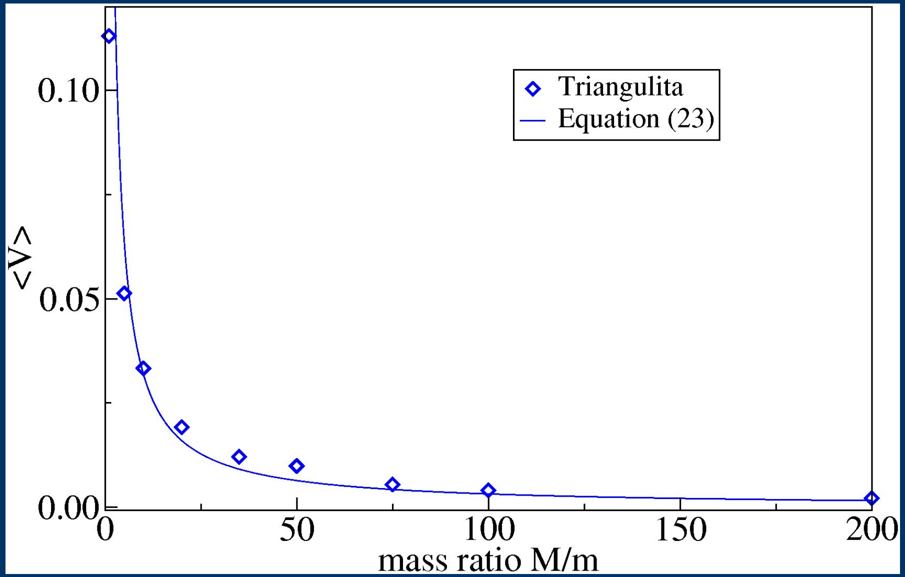
T2
T1



Triangula
Triangulita
Equation (23)
Equation (22)
<V>
0.02
0.01
0.00
-0.01
-2
-1
0
1
2
T1-T2



T2
T1



Triangulita
Equation (23)
<V>
0.10
0.05
0.00
0
50
100
150
200
mass ratio M/m

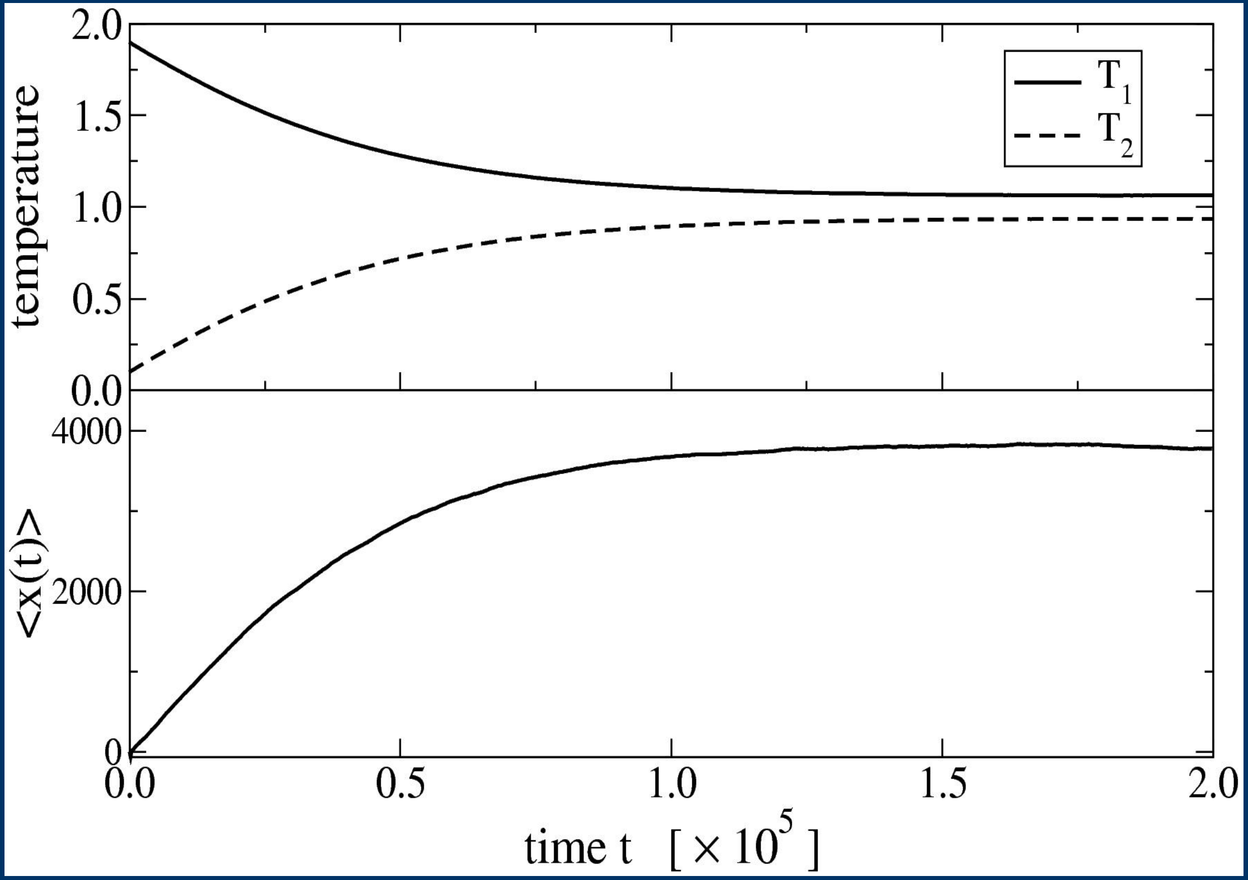
Heat Conduction

Fourier's Law

$$\dot{Q}_{1\to 2} = \kappa(T_1 - T_2)$$

Conductivity

$$\kappa = \frac{k_B \gamma_1 \gamma_2}{M(\gamma_1 + \gamma_2)}$$



Linear Irreversible thermodynamics

$$J_A = L_{AA} X_A + L_{AB} X_B$$
$$J_B = L_{BA} X_A + L_{BB} X_B$$

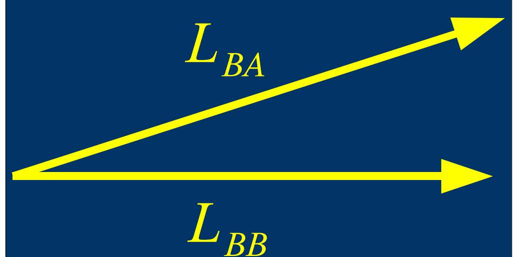
Transport Coefficients

$$L_{AA} = \frac{T}{\gamma_1 + \gamma_2} \qquad L_{BB} = \kappa T^2$$
$$L_{AB} = L_{BA} = \rho_1 \rho_2 (1 - \sin^2\theta_0) \frac{m}{M} \sqrt{\frac{\pi k_B}{2\mathrm{m}}} \frac{T^{3/2}}{[2\rho_1 + \rho_2(1 + \sin\theta_0)]^2}$$

Thermodynamics Force

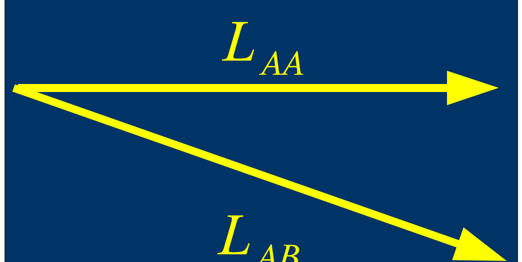
Flux

$$X_A = \frac{F}{T} = 0$$
$$X_B = \frac{\Delta T}{T^2} \neq 0$$



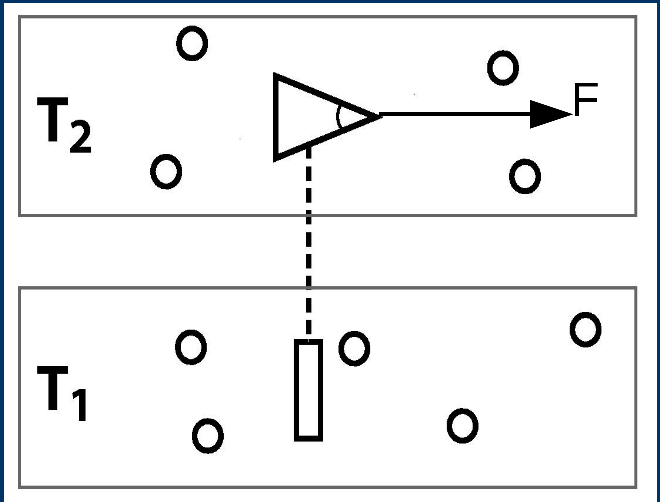
$$J_A = \langle V \rangle$$
$$J_B = \dot{Q}$$

$$X_A = \frac{F}{T} \neq 0$$
$$X_B = \frac{\Delta T}{T^2} = 0$$



$$J_A = \langle V \rangle$$
$$J_B = \dot{Q}$$

Onsager Symmetry



Joule heat $$\dot{Q}_i^J = \frac{\gamma_i}{(\gamma_1+\gamma_2)^2} F^2$$

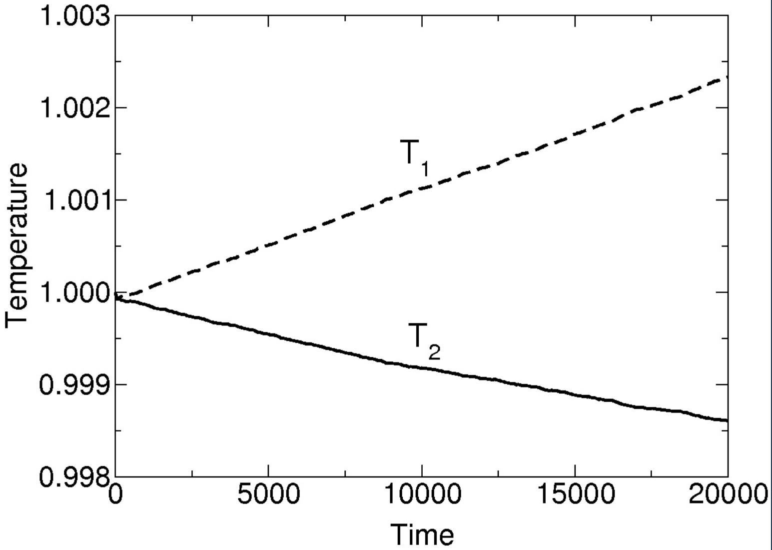
$$\dot{Q}_{1\to 2} = \rho_1 \rho_2 (1-\sin^2\theta_0) \frac{m}{M} \sqrt{\frac{\pi k_B T}{2\mathrm{m}}} \frac{F}{[2\rho_1 + \rho_2(1+\sin\theta_0)]^2}$$

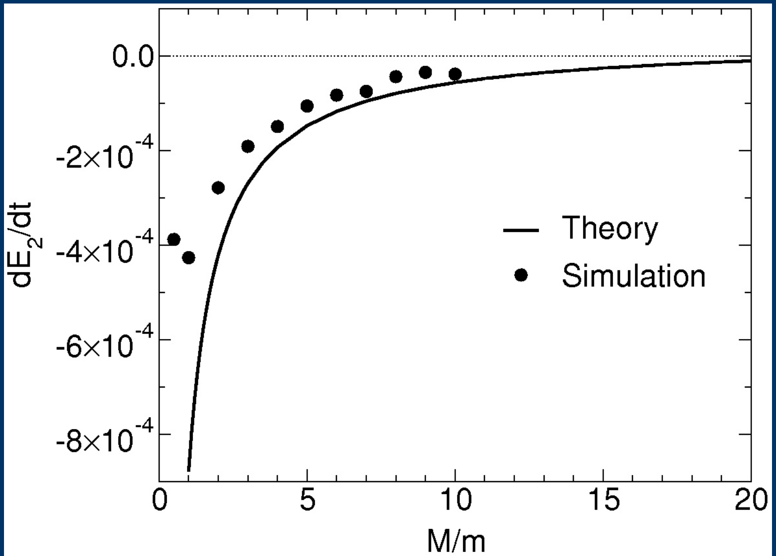
The direction of heat transfer is determined by the direction of force.

$$\dot{Q}_1 = \dot{Q}_1^J - \dot{Q}_{1\to 2}$$

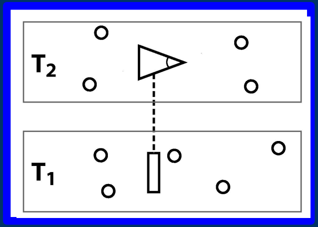
$$\dot{Q}_2 = \dot{Q}_2^J + \dot{Q}_{1\to 2}$$

Using a small force F, $\dot{Q}_i < 0$ is possible.

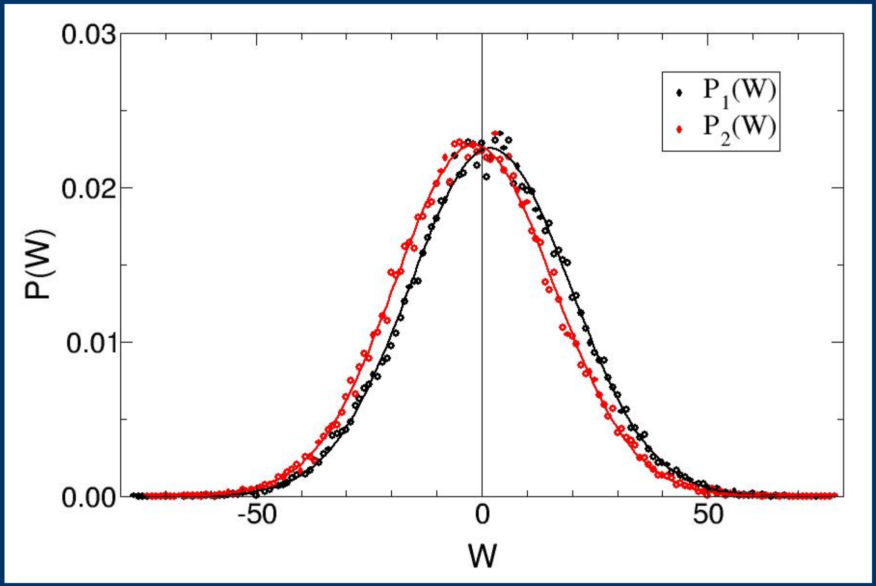




Simulation $F = -0.007\,,\ \ \dfrac{m}{M} = 2$



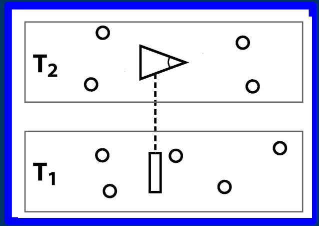
Work done by triangle and bar

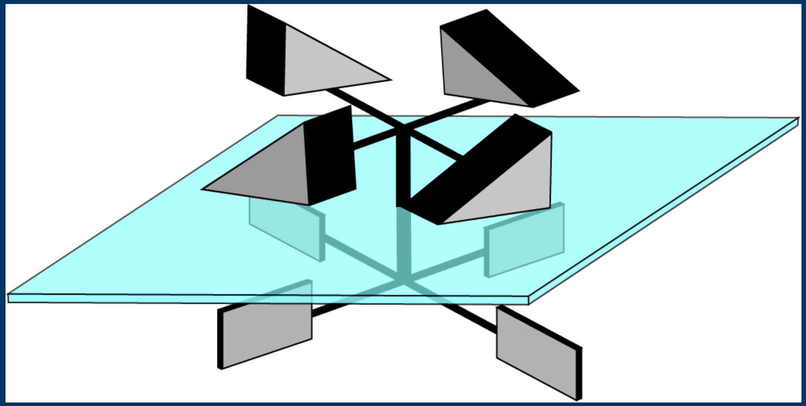


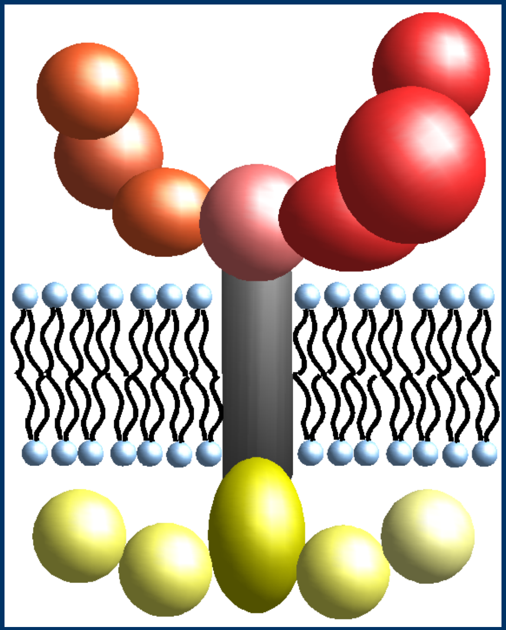
$$\langle W_1 \rangle + \langle W_2 \rangle = \Delta U = \frac{F^2}{\gamma_1 + \gamma_2} t$$

$$\langle W_1 \rangle = \frac{\gamma_1 F^2}{\gamma_1 + \gamma_2} t + Q_{1\to 2} t$$

$$\langle W_2 \rangle = \frac{\gamma_2 F^2}{\gamma_1 + \gamma_2} t - Q_{1\to 2} t$$







Conclusions

- A novel microscopic motor powered by the rectification of thermal fluctuations is proposed
- Hard-disk molecular dynamics simulation confirms that the model indeed generates work by rectifying thermal fluctuation of gas molecules.
- An analytical theory correctly predicts velocity of the motor and heat conduction.
- A novel microscopic refrigerator in which thermal fluctuations themselves are harnessed to reduce the thermal jitter in one part of the system is predicted based on Onsager symmetry.
- Computer simulation confirms the Brownian refrigerator.
- Both Brownian motors and Brownian refrigerators are genuine microscopic machines (no macroscopic equivalent).