Project: Debye Model of Heat Capacity

Based on Debye theory, the heat capacity of a solid at temperature T is given by

$$C_V = 9kN\left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \frac{x^4 \mathrm{e}^x}{(\mathrm{e}^x - 1)^2} \mathrm{d}x$$

where θ_D is the Debye temperature, N is the number of atoms, and k is Boltzmann's constant. Plot the molar specific heat of copper ($\theta_D = 309\,K$) from $T = 0\,K$ to $T = 1083\,K$.

Tasks

1. Rewrite the equation using dimensionless quantities.
2. Find a way to evaluate the integrand at $x = 0$.
3. Evaluate the integral for various temperatures.
4. Plot the "normalized" heat capacity as a function of "normalized" temperature.
5. Discuss if the curve is universal or depends on the material under consideration.

Required numerical methods

1. Simplification of mathematical expression suitable for numerical evaluation.
2. Numerical integration of improper integrals.
3. Visualization of the numerical data.

[Garcia: *Numerical Methods for Physics*, 2nd edition, Chapter 10, Exercise 16.]