Suggested Project
Classical Scattering

A particle of mass m is scattered by another particle of mass m_2 with impact parameter b. We The interaction between the particles is given by a spherical potential $U(r) = \frac{k}{r}\mathrm{e}^{-r/a}$ where r is the distance between the particles. The parameter a and k are both positive. The general expression of the scattering angle is given by $\theta = \pi - 2\phi$ where

$$\phi = \int_{r_m}^{\infty} \frac{b}{r^2} \frac{1}{\sqrt{1 - \frac{b}{r^2} - \frac{U(r)}{E}}} \mathrm{d}r$$

where E is the energy of the system and the lower integral limit is the root of

$$1 - \frac{b}{r^2} - \frac{U(r)}{E} = 0$$

Tasks

1. Normalize the energy and distance appropriately so that we don't have to specify the value of k and a.
2. Find the closest approach r_m for energy $E = 1$ in the normalized units.
2. Find a scattering angle (in the laboratory frame) by integrating the analytical expression given above. Plot the angle as the function of the impact parameter b.
3. Find the same quantity by solving Newton's equation of motion directly.
4. Plot the shadow cone for the given energy E.

Required numerical methods

1. Root finding. (Chapter 3)
2. Numerical integration of improper kind integrals. (Chapter 2)
3. ODE: Initial value problem. (Chapter 4)