Suggested Project
Frequency of Classical Oscillation

A particle of mass m is moving along a straight line in a periodic potential field $U(x) = U_0 \left[\sin\left(\frac{2\pi x}{L}\right) - \frac{1}{4}\sin\left(\frac{4\pi x}{L}\right)\right]$ where x is the position of the particle, L the periodicity of the potential, and U_0 the strength of the potential. For appropriate energy E, the particle oscillates between two turning points x_1 and x_2.

Tasks

1. Find appropriate normalization of energy E and distance x so that we don't have to specify the values of L and U_0. With the normalization you use, what is the unit time?

2. Find the range of energy E for which the particle is bound and oscillates between two points.

3. Find the turning points for three different values of energy E.

4. Find the frequency of the oscillation for each energy E using the formula for the period:
$$T = 2\int_{x_1}^{x_2} \frac{1}{v(x)}\, \mathrm{d}x$$
where x_1 and x_2 are the turning points obtained in part 3, and $v(x)$ the speed of the particle.

5. Find the frequency of the oscillation by solving Newton equation numerically.

6. Compare the results of two different numerical methods.

Required numerical methods

1. Root finding. (Chapter 3)

2. Numerical integration of improper integrals. (Chapter 2)

3. ODE: Initial value problem. (Chapter 4)